HALF YEARLY EXAMINATION, 2024-25

MATHEMATICS

Time – 3:00 Hrs. **Class – XII** **M.M. : 80**

Date – 14.09.2024 (Saturday)

Name of the student _____Section _____

GENERAL INSTRUCTIONS:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there is some internal choice in some questions.
2. Section A has 18 MCQ's and 02 Assertion Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA) questions of 2 marks each.
4. Section C has 6 Short Answer (SA) questions of 3 marks each.
5. Section D has 4 Long Answer (LA) questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (04 marks each) with sub parts.

SECTION –A

This section comprises of multiple choice questions of 1 mark each

Q1 If $f: [1, \infty) \to [2, \infty)$ is given by $f(x) = x + \frac{1}{x}$, then $f^{-1}(x)$ is

a) $\frac{x+\sqrt{x^2-4}}{2}$ b) $\frac{x}{1+x^2}$ c) $\frac{x-\sqrt{x^2-4}}{2}$ d) $1 + \sqrt{x^2-4}$

Q2 If a relation R is defined on the set Z of integers as follows: $(a, b) \in R \Leftrightarrow a^2 + b^2 = 25$. Then, domain (R) is

a) {3,4,5} b) {0,3,4,5} c) {0, ±3, ±4, ±5} d) none of these

Q3 The relation $R = \{(1,1), (2,2), (3,3)\}$ on the set {1,2,3} is

a) Symmetric only b) reflexive only c) an equivalence relation d) transitive only

Q4 If $tan^{-1}\left\{\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right\} = \alpha$, then $x^2 =$

a) $sin2\alpha$ b) $sin\alpha$ c) $cos2\alpha$ d) $cos\alpha$

Q5 If $4cos^{-1}x + sin^{-1}x = \pi$, then the value of x is

a) $\frac{3}{2}$ b) $\frac{1}{\sqrt{2}}$ c) $\frac{\sqrt{3}}{2}$ d) $\frac{2}{\sqrt{3}}$

Q6 $sin[cot^{-1}\{tan(cos^{-1}x)\}]$ is equal to

a) x b) $\sqrt{1-x^2}$ c) $\frac{1}{x}$ d) none of these

Q7 The value of $\begin{vmatrix} 5^2 & 5^3 & 5^4 \\ 5^3 & 5^4 & 5^5 \\ 5^4 & 5^5 & 5^6 \end{vmatrix}$ is

a) 5^2 b) 0 c) 5^{13} d) 5^9

Q8 If A is a matrix of order 3 × 3 is such that $|A| = 4$, then the value of $|2A|$ is

a) 32 b) 8 c) 0 d) cannot able be find

Q9 If the matrix $\begin{bmatrix} 5-x & x+1 \\ 2 & 4 \end{bmatrix}$ is a singular matrix, then what will be the value of x

a) 1 b) 4 c) 0 d) 3

Q10 If A is a square matrix such that $A^2 = I$, then A^{-1} is equal to

a) $A + I$ b) A c) 0 d) $2A$

Q11 If A and B are symmetric matrices, then ABA is

a) Symmetric matrix b) skew – symmetric matrix

c) diagonal matrix d) scalar matrix

Q12 The number of all possible matrices of order 3 x 3 with each entry 0 or 1 is

a) 27 b) 18 c) 81 d) 512

Q13 Let $f(x) = |x| + |x - 1|$, then

a) $f(x)$ is continuous at x = 0 as well as at x = 1

b) $f(x)$ is continuous at x = 0 but not at x = 1

c) $f(x)$ is continuous at x = 1 but not at x = 0

d) none of these

Q14 Differential coefficient of $sec(tan^{-1}x)$ is

a) $\frac{x}{1+x^2}$ b) $x\sqrt{1 + x^2}$ c) $\frac{1}{\sqrt{1+x^2}}$ d) $\frac{x}{\sqrt{1+x^2}}$

Q15 If $x^y = e^{x-y}$, then $\frac{dy}{dx}$ is

a) $\frac{1+x}{1+logx}$ b) $\frac{1-logx}{1+logx}$ c) not defined d) $\frac{logx}{(1+logx)^2}$

Q16 If $x = acos\, nt - bsinnt$, then $\frac{d^2y}{dx^2}$ is

a) n^2x b) $-n^2x$ c) $-nx$ d) nx

Q17 If $x = t^2, y = t^3$, then $\frac{d^2y}{dx^2}$ is

a) $\frac{3}{2}$ b) $\frac{3}{4t}$ c) $\frac{3}{2t}$ d) $\frac{3t}{2}$

Q18 The volume of a sphere is increasing at the rate of 0.2cm/sec. The rate at which the volume of the sphere increases when the radius is 15cm is

a) $12\pi cm^3/sec$ b) $180\pi cm^3/sec$ c) $225\,\pi cm^3/sec$ d) $3\pi cm^3/sec$

Assertion – Reason Based Questions

In the following questions, a statement of Assertion (A) is followed by a statement of Reason(R). Choose the correct answer out of the following choices.

(a)Both (A) and (B) are true and (R) is the correct explanation of (A).

(b) Both (A) and (B) are true and (R) is not the correct explanation of (A).

(c) (A) is true and (R) is false

(d) (A) is false and (R) is true

Q19 **Assertion (A):** The absolute maximum value of $y = x^3 - 3x + 2$ in $0 \leq x \leq 2$ is 4

Reason (R): Slope of the tangent to the curve $y = x^3 - x + 1$ is 14 at the point whose x – coordinate is 2.

Q20 **Assertion (A):** Let the function $f(x) = x^7 + 14x^5 + 16x^3 + 30x - 560$. The equation f(x) = 0 has 7 real roots.

Reason (R): Above function f(x) is an increasing function in x.

SECTION – B

This section comprises of very short answer type questions of 2 marks each

Q21 Let $A = \{1,2,3\}, B = \{4,5,6,7\}$ and let $f = \{(1,4), (2,5), (3,6)\}$ be a function from A to B. State whether f is one – one, onto or not and why?

OR

If the function $f: R \to R$ defined by $f(x) = 3x - 4$ is bijective, then find its inverse.

Q22 Write the value of $\begin{vmatrix} a + ib & c + id \\ -c + id & a - ib \end{vmatrix}$

Q23 If $A = [a_{ij}]$ is a square matrix such that$a_{ij} = i^2 - j^2$, then write whether A is symmetric or skew – symmetric.

OR

Find the equation of line joining (1,2) and (3,6) using determinants.

Q24 If $y = sin^{-1}\left(\frac{2x}{1+x^2}\right)$, write the value of $\frac{dy}{dx}$ for $x > 1$.

Q25 The radius of a balloon is increasing at the rate of 10cm/sec. At what rate is the surface area of the balloon increasing when the radius is 15cm?

SECTION – C

This section comprises of short answer type questions of 3 marks each

Q26 If $sin\left(sin^{-1}\frac{1}{5} + cos^{-1}x\right) = 1$, then find the value of x

Q27 Solve $\begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix}$

Q28 If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ and $I\ is\ identity\ matrix$, then find λ so that $A^2 - 5A - \lambda I = 0$

OR

Examine the consistency of the following system of equations: x+2y=2 and 2x+3y=3

Q29 If $y = \sqrt{\frac{1-x}{1+x}}$, prove that $(1 - x^2)\frac{dy}{dx} + y = 0$

Q30 A water tank has the slope of an inverted right circular cone with its axis vertical and vertex lower most. Its semi – vertical angle is $tan^{-1}(0.5)$. Water is poured into it at a constant rate of 5 cubic metre per hour. Find the rate at which the level of the water is rising at the instant when the depth of water in the tank is 4m.

OR

A particle moves along the curve$6y = x^3 + 2$. Find the points on the curve at which the y – coordinate is changing 8 times as fast as the x – coordinate.

Q31 Find the intervals in which the function f given by $f(x) = \frac{4sinx - 2x - xcosx}{2+cosx}, 0 \le x \le 2\pi$ is increasing and decreasing.

SECTION – D

This section comprises of long answer type questions of 5 marks each

Q32 Show that the relation R on the set$A = \{x \in Z: 0 \le x \le 12\}$, given by

$R = \{(a, b): |a - b| is\ a\ multiple\ of\ 4\}$ is an equivalence relation. Find the set of all elements related to 1.

Q33 Determine the product $\begin{bmatrix}-4 & 4 & 4\\ -7 & 1 & 3\\ 5 & -3 & -1\end{bmatrix}\begin{bmatrix}1 & -1 & 1\\ 1 & -2 & -2\\ 2 & 1 & 3\end{bmatrix}$ and using it to solve the system of linear equations: $x - y + z = 4, x - 2y - 2z = 9, 2x + y + 3z = 1$

OR

If $A = \begin{bmatrix}0 & -tan\frac{\alpha}{2}\\ tan\frac{\alpha}{2} & 0\end{bmatrix}$ and I is the identity of order 2, show that

$I + A = (I - A)\begin{bmatrix}cos\alpha & -sin\alpha\\ sin\alpha & cos\alpha\end{bmatrix}$

Q34 Determine the values of a, b and c for which the function $f(x) = \begin{cases}\frac{sin(a+1)x+sinx}{x}, for\ x < 0\\ c, for\ x = 0\\ \frac{\sqrt{x+bx^2}-\sqrt{x}}{bx^{3/2}}, for\ x > 0\end{cases}$ is continuous at x = 0

Q35 Show that the semi – vertical angle of a right circular cone of maximum volume and given slant height is $tan^{-1}\sqrt{2}$

OR

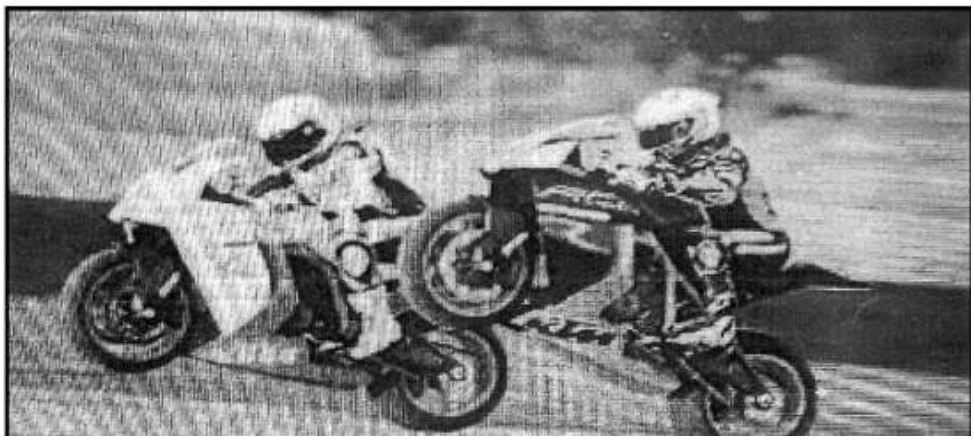
Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$. Also, find the maximum volume.

SECTION – E

This section comprises of 3 cases – study/passage bases questions of 4 marks each with sub – parts. The first two case – study questions have three parts (a), (b) and (c) of 1,1 and 2respectively. The third case – study question has two sub – parts of two marks each.

Q36 An organization conducted bike race under two different categories boys and girls. Totally there are 250 participants. Among all of them finally three from Category – 1 and two from Category – 2 were selected for the final race. Mohan forms two sets Band G with these participants for his college project.

Let = $\{b_1, b_2, b_3\}, G = \{g_1, g_2\}$, where B represents the set of boys selected and G the set of girls who were selected for the final race.



Based on the above information, answer the following questions:

(a) How many relations are possible from B to G?

(b) Among all the possible relations from B to G, how many functions can be formed from B to G?

(c) Let $R: B \to B$ be defined by$R = \{(x, y): x \text{ and } y \text{ are the students of same sex}\}$. Check if R is a equivalence relation.

OR

(c) A function $f: B \to G$ be defined by $f = \{(b_1, g_1), (b_2, g_2), (b_3, g_1)\}$. Check if f is bijective. Justify your answer.

Q37 Mr. Sudhir and Mr. Amit decided to reward their kids on the basis of good habits practiced during their summer holidays namely daily yoga(x) and regular studies(y). Mr. Sudhir decided to give 100 chocolates for the two values to his two boys and one girl child. Mr. Amit decided to give 60 chocolates for the two values to his one boy and one girl child.

Based on the above information, answer the following questions:

(a) Convert the given above situation into a matrix equation of the form AX=B

(b) Find $|A|$

(c) Find A^{-1}

OR

Determine $P = A^2 - 5A$

Q38 A square piece of tin sheet measuring 24 cm x 24 cm is to be made into a box without top by cutting a square from each corner and folding up the flaps to form the box.

Based on the above information, answer the following questions:

(a) Find the volume of the box

(b) Find the differentiation of the volume.

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SECTION -A

Q1. If R = {(a, b): 2 divides (a – b)} be the equivalence relation on the set A = {0,1,2,3,4,5} then the equivalence class [0] is given by **1**

(a) {0,2,4} (b) {1,3,5} (c) {2,4} (d) {3,5}

Q2. If a relation R on the set A = {3,4,5} be defined by R={(3,4),(4,3),(3,3),(4,4)} then R is **1**

(a) Reflexive (b) Symmetric (c) Transitive (d) Symmetric and transitive

Q3. Let f: R→R be defined by f(x) =1/x ,∀ x ∈ R. Then f is **1**

(a) one-one (b) onto (c) Bijective (d) f is not defined

Q4. If $\sin^{-1}x - \cos^{-1}x = \pi/6$, then x is equal to: **1**

(a) 1/2 (b) -1/2 (c) $\frac{\sqrt{3}}{2}$ (d) $-\frac{\sqrt{3}}{2}$

Q5. Evaluate : $\tan^{-1}(tan 5\pi/6)$ **1**

(a) $\pi/6$ (b) $-\pi/6$ (c) $\frac{\pi}{3}$ (d) $-\frac{5\pi}{6}$

Q6. Find the principal value of $\tan^{-1}\sqrt{3} + \cot^{-1}(-\sqrt{3})$ is **1**

(a) $\pi/6$ (b) $7\pi/6$ (c) $\frac{5\pi}{3}$ (d) $-\frac{5\pi}{6}$

Q7. The area of a triangle with vertices (2, –6), (5 , 4) and (K, 4) is 35 sq. units then k is **1**

a)12 b) – 2 c) –12 , –2 d)12 , –2

Q8. If A and B are invertible matrices of order 3, $|A| = 2$ and $|(AB)^{-1}| = -1/6$, Find $|B|$ **1**

a) – 1/3 b) 3 c) –1/12 d) –3

Q9. If a matrix A is both symmetric and skew –symmetric , then A is necessarily a **1**

a) diagonal matrix b) zero square matrix c) square matrix d) Identity matrix

Q10. The number of all possible matrices of order 2×3 with entry 1 or 2 **1**

a) 16 b) 64 c) 6 d) 24

Q11. The function f(x) =[x] is continuous at **1**

a) 4 b) –2 c) 1 d) 1.5

Q12. If $x = t^2$ and $y = t^3$ then $\frac{d^2y}{dx^2}$ is equal to **1**

(a) $\frac{3}{2}$ (b) $\frac{3}{2t}$ (c) $\frac{3}{4t}$ (d) $-\frac{3t}{2}$

Q13. Derivative of x^2 with respect to x^3 is **1**

(a) $\frac{1}{x}$ (b) $\frac{2}{3x}$ (c) $\frac{2}{3}$ (d) $-\frac{3x}{2}$

Q14. The derivative of f(tanx) w.r.t g(secx) at $x = \pi/4$, where $f^1(1) = 2$ and $g^1(\sqrt{2}) = 4$ is **1**

(a)$1/\sqrt{2}$ (b)$\sqrt{2}$ (c) 0 (d)1

Q15. Function $f(x)=logx$ is increasing on R, if **1**

(a) $0<x<1$ (b) $x>1$ (c) $x<1$ (d) $x>0$

Q16 The Maximum and Minimum values of the function $|\sin 4x+3|$ are **1**

(a) 1, 2 (b) 4, 2 (c) 2, 4 (d) 1, 1

Q17. If $x+y=10$, then the maximum value of xy is **1**

(a) 5 (b) 20 (c) 25 (d) None of these

Q18. Which of the following function is decreasing on $(0, \pi/2)$ **1**

(a) tan2x (b) cosx (c) cos 3x (d) none of these

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

Q19. Assertion (A) - The points A(a, b+c), B(b,c+a) and C(c, a+b) are collinear: **1**

Reason (R) - Area of triangle with three collinear points is zero

Q20. Assertion (A) - If A is a 3x3 non–singular matrix then $A^{-1} = adjA / |A|$. **1**

Reason (R) - If A and B both are invertible matrices such that B is inverse of A, then AB=BA=I

SECTION B

Q21 For the following matrices A and B, verify that (AB)' = B'A'. **2**

$A = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}$, $B = [-1 \quad 2 \quad 1]$.

Q22 Find the value of x ,y, z and w from the following matrix equation: **2**

$$\begin{bmatrix} x-y & z \\ 2x-y & w \end{bmatrix} = \begin{bmatrix} -1 & 4 \\ 0 & 5 \end{bmatrix}$$

Q23 Find the interval in which the function $F(x) = 2x^3 - 9x^2 + 12x + 15$ is decreasing **2**

Q24 If $x=a\cos^3\theta$, $y=a\sin^3\theta$, then find the value of $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{6}$. **2**

Q25 The relation S in the set R of real numbers, defined as **2**

$S = \{(a, b) : a, b \in R \text{ and } a \leq b^3\}$, verify for reflexive, symmetric.

SECTION - C

Q26. Find the interval in which the function f(x) = $-x^3+3x^2+9x-27$ is increasing and decreasing. **3**

OR

Find the least value of k such that the function f given by f(x) = x^2 + kx + 1 is strictly increasing on (1, 2).

Q27. Find the minimum value of the function $2\cos 2x - \cos 4x$ in $0 \le x \le \pi$. **3**

Q28. Evaluate : $\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)$ **3**

Q29 Express the following matrix as the sum of a symmetric matrix and a skew symmetric matrix. **3**

$$A = \begin{bmatrix} 1 & 3 & 5 \\ -6 & 8 & 3 \\ 4 & 6 & 5 \end{bmatrix}$$

Q30 If $x^p y^q = (x+y)^{p+q}$, then dy/dx **3**

OR

If $y = Ae^{5x} + Be^{-5x}$, then prove that $d^2y/dx^2 = 25y$

Q31 If matrix $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$, then show that $A^2 - 4A + I = 0$ and using it find A^{-1}. **3**

SECTION – D

Q32. Find the value of a for which the following function is continuous at x = 0. **5**

$$F(x) = \begin{cases} a \sin \frac{\pi}{2}(x+1), & x \le 0 \\ \frac{\tan x - \sin x}{x^3}, & x > 0. \end{cases}$$

Q33. **5**

If $= \begin{pmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{pmatrix}$ find A^{-1}. Using A^{-1} solve the following system of equations:

2x – 3y + 5z = 16; 3x + 2y – 4z = – 4; x + y – 2z = –3

OR

Find AB where $A = \begin{pmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{pmatrix}$. Using the result

solve: x – y = 3 ; 2x + 3y + 4z = 17 ; y + 2z = 7.

Q34. Let A and B be sets. f : A × B→B × A such that f (a, b) = (b, a). State whether the function is one-one, onto or bijective. Justify your answer. **5**

Q35. Show that the right circular cylinder of given surface and maximum volume is such that its height is equal to the diameter of the base. **5**

OR

a) A ladder 5m long is leaning against a wall, the bottom of the ladder is pulled along the ground, away from the wall, at the rate of 2 cm/sec. How fast is the height on the wall decreasing when the foot of the ladder is 4m away from the wall? (3 marks)

b) The volume of a cube is increasing at the rate of 9 cm^3/s. How fast is its surface area increasing when the length of an edge is 10 cm? (2 marks)

SECTION-E (CASE BASED QUESTIONS)

Q36. Sherlin and Dhanraj are playing Ludo at home during Covid-19. While rolling the dice, Sherlin's sister Raji observed and noted the possible outcomes of the throw every time belongs to set {1, 2, 3, 4, 5, 6}. Let A be the set of players while B be the set of all possible outcomes. A = {S, D}, B = {1, 2, 3, 4, 5, 6} **(1+1+2)**

a) Let $R: B \rightarrow B$ be defined by R = $\{(x,y): y \text{ is divisible by } x\}$ .Verify for transitive.

b) Raji wants to know the number of relations from A to B. How many number of relations are possible?

c) Raji wants to know the number of functions from A to B. How many numbers of functions are possible?

OR

c) Let R be a relation on B defined by R ={(1,2), (2,2), (1,3), (3,4), (3,1), (4,3), (5,5)}. Then verify R is Symmetric, Reflexive and Transitive .

Q37. Three schools DPS, OPJS and KVS decided to organize a fair for collecting money for helping the flood victims. They sold handmade fans, mats and plates from recycled material at a cost of Rs. 25, Rs.100 and Rs. 50 each respectively. The numbers of articles sold are given as **(1+1+2)**

School /Article	DPS	OPJS	KVS
Handmade fans	40	25	35
Mats	50	40	50
Plates	20	30	40

Based on the information given above, answer the following questions:

a) What is the total money (in Rupees) collected by the school DPS?

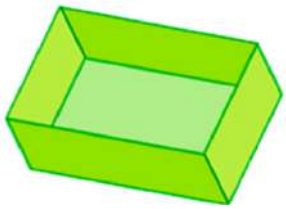
b) What is the total amount of money (in Rs.) collected by schools OPJS and KVS?

c) If the number of handmade fans and plates are interchanged for all the schools, then what is the total money collected by all schools?

OR

c) How many articles (in total) are sold by three schools?

Q38. A farmer wants to construct a small tank with rectangular base and rectangular sides, open at the top is to be constructed so that its depth is 2 m and volume is 8 m3. The cost of building tank is Rs. 70 per sq metre for the base and Rs. 45 per sq metre for sides. **(1+1+2)**



Based on the above information, answer the following questions: (Attempt any four)

a) Find a function that models the cost of the box.

b) Find the length and breadth of the box for which the cost of the box is minimum.

c) Find the cost of least expensive tank.

OR

c) If the volume is doubled, then find the increased cost of tank.