

(17) Solve:  $4 \{n\} = x + [n]$

$$\text{Sopn: } \{n\} = \frac{2[n]}{3} \quad \text{--- (1)}$$

$$\therefore 0 \leq \frac{2[n]}{3} < 1 \Rightarrow 0 \leq [n] < \frac{3}{2} \Rightarrow [n] = 0, 1$$

If  $[n] = 0$ ,  $\{n\} = 0$  & if  $[n] = 1$ ,  $\{n\} = \frac{2}{3}$

$$\therefore n = \frac{5}{3}$$

$$\text{So, } \boxed{n = 0, \frac{5}{3}}$$

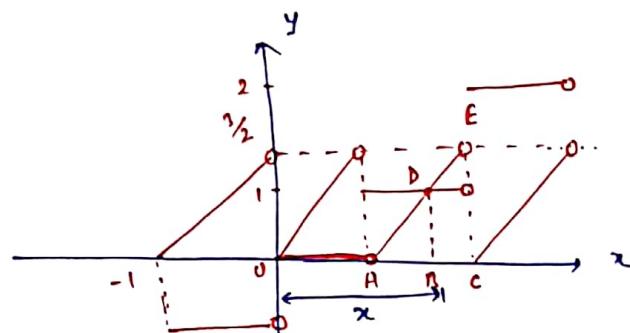
We can solve it graphically,

in  $\triangle ABD \& \triangle ACE$

$$\frac{AB}{EC} = \frac{AD}{AE}$$

$$\text{i.e. } AB = \frac{2}{3}$$

$$\therefore n = 1 + \frac{2}{3} = \frac{5}{3} \text{ or } n = 0.$$



$$\text{i.e. } \boxed{n = 0, \frac{5}{3}}.$$

(18) A, B are 2 decimal numbers &  $x = [A+B]_{\max}, B = [A]+[B]$ . Find que. eq? having roots  $\alpha + \beta$ .

Sopn: Here  $0 < A, B < 1$

$$\text{Also, } \left. \begin{array}{l} 0 < A < 1 \\ 0 < B < 1 \end{array} \right\} \Rightarrow 0 < A+B < 2$$

$$\therefore \alpha = 1, \beta = 0+0 = 0$$

$$\text{So, eq: } \boxed{\sqrt{n} - x = 0}$$

(19.) If  $f(n)$  be a polynomial func. satisfying  $f(n)f(1/n) = f(n) + f(1/n)$  &  $f(4) = 65$ , find  $f(5)$ .

→ Students can Try this.

### ODD AND EVEN FUNCTIONS:-

ODD FUNCTION :- func. which satisfies  $f(-n) = -f(n)$ , known as odd func. ex:  $x^{2n+1}, (\sin x)^{2n+1}$  etc

\* Graph of such func. are symmetrical about origin.

\* If  $f(x)$  is rotated by  $180^\circ$  about the origin, the graph gives same appearance as original.

EVEN FUNCTIONS!:-

functions which satisfy  $f(-n) = f(n)$ . ex:  $n^2$ , even etc.

\* If graph is rotated by  $180^\circ$  about y-axis, it gives same appearance as original.

Note:-

$f(n)$	$g(n)$	$f(n) \pm g(n)$	$f(n) \times g(n)$	$f(n)/g(n); g \neq 0$
odd	odd	odd	even	even
odd	even	Neither	odd	odd
even	odd	Neither	odd	odd
even	even	even	even	even.

\* Derivative of an odd  $f^n$  is an even function & vice-versa.

\* Every function can be expressed as the sum of an even and an odd  $f^n$ .

(20.) Check the following functions are odd or, even.

$$(a) f(n) = \frac{n \sin n}{[\frac{n}{\pi}] + \frac{1}{2}}, \quad [ ] \rightarrow \text{G.P.F.}$$

$$\text{Sol: } \therefore f(-n) = \frac{n \sin n}{[-\frac{n}{\pi}] + \frac{1}{2}}$$

$$\therefore \left[ -\frac{n}{\pi} \right] \Rightarrow \begin{cases} -[\frac{n}{\pi}], & \text{if } \frac{n}{\pi} = I \text{ i.e. } n = n\pi, n \in \mathbb{Z} \\ -[\frac{n}{\pi}] - 1, & \text{if } \frac{n}{\pi} \neq \text{int i.e. } n \neq n\pi, n \in \mathbb{Z} \end{cases}$$

$$\text{If } n = n\pi, \quad f(n) = 0 \quad \& \quad f(-n) = 0 = -f(n)$$

$$\text{If } n \neq n\pi, \quad f(-n) = \frac{n \sin n}{-([\frac{n}{\pi}] + \frac{1}{2})} = -f(n)$$

Thus,  $f(-n) = -f(n)$  in both cases.

∴ odd func.

$$(b) f(n) = 2 \tan^{-1}(e^n) - \pi_2$$

Soln:-  $\therefore f(-n) = 2 \tan^{-1}(e^{-n}) - \pi_2 = 2 \cot^{-1}(e^n) - \pi_2$   
 $= 2(\pi_2 - \tan^{-1}(e^n)) - \pi_2$   
 $= \pi_2 - 2 \tan^{-1}(e^n) = -f(n)$

So, odd function.

JEE  
(c)  $f(n) = \log(x + \sqrt{x^2+1})$ .

Soln:-  $\therefore f(-n) = \log(-x + \sqrt{x^2+1}) = \log\left\{\frac{\sqrt{x^2+1} - x}{\sqrt{x^2+1} + x} \times \frac{\sqrt{x^2+1} + x}{\sqrt{x^2+1} + x}\right\}$   
 $= \log\left\{\frac{x^2+1 - x^2}{\sqrt{x^2+1} + x}\right\}$   
 $= -\log(x + \sqrt{x^2+1})$   
i.e.  $f(-n) = -f(n) \Rightarrow$  odd f?

JEE  
(d) If  $f: (-\pi_2, \pi_2) \rightarrow \mathbb{R}$ ,  $f(x) = (\log(\sec x + \tan x))^n$

→ students must try this.

(e) Let  $f'(n) = \left[ \frac{f(n) + f(-n)}{g(n) + g(-n)} \right]^n$ , show that  $f'(n)$  is even if? when 'n' is even & odd when 'n' is odd.

Soln:- Let  $k(n) = f(n) + f(-n) \Rightarrow k(-n) = f(-n) + f(n) = k(n)$

&  $h(n) = g(n) + g(-n) \Rightarrow h(-n) = g(-n) + g(n) = -h(n)$

So,  $k(n)$  is an even f? &  $h(n)$  is an odd f?

$$\therefore f'(n) = \left[ \frac{k(n)}{h(n)} \right]^n$$

i.e.  $f'(-n) = (-1)^n \left[ \frac{k(n)}{h(n)} \right]^n = (-1)^n f'(n)$

$\therefore f'(-n) \Rightarrow$   $f'(n)$ , n even  
 $-f'(n)$ , n odd.

(21.) If  $f(x+y) = f(x) \cdot f(y)$  for  $x, y$  and  $f(0) \neq 0$ , then prove that the func

$$g(x) = \frac{f(x)}{1+(f(x))^2} \text{ is even.}$$

Say ":- Putting,  $x=y=0$ ,  $f(0)=1$

Now, putting  $y=-x$ ,  $f(0) = f(x) f(-x) \Rightarrow f(-x) = 1/f(x)$

$$\therefore g(x) = \frac{f(x)}{1+(f(x))^2} \Rightarrow g(-x) = \frac{f(-x)}{1+(f(-x))^2}$$

$$\text{Hence, } g(-x) = \frac{1/f(x)}{1+1/(f(x))^2} = \frac{f(x)}{1+(f(x))^2} = g(x)$$

i.e.  $g(x)$  is an even fn.

SEE

(22.) If 'f' is an even fn defd. on the interval  $(-s, s)$ , then four real values of 'x' satisfying the eqn.  $f(x) = f\left(\frac{x+1}{x+2}\right)$  are ...

→ students can try this.

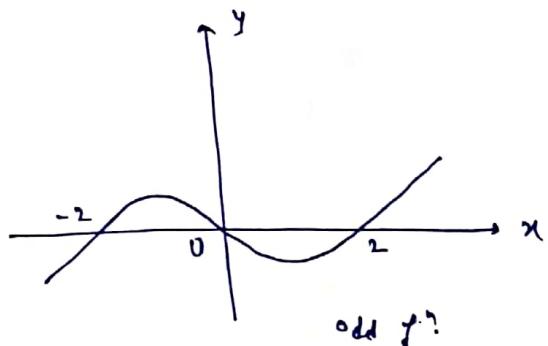
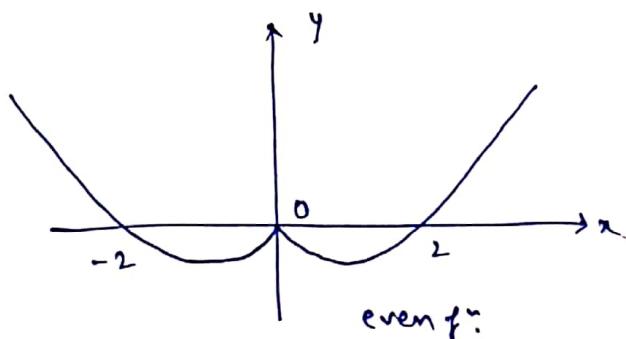
(23.) Given that  $f(x) = x^2 - 2x$ ,  $x > 0$ . Find the exp extension of  $f(x)$  so that it is (a) even in  $\mathbb{R}$  (b) odd in  $\mathbb{R}$ .

Say ":- (a)  $f(x)$  is even if  $f(x) = f(-x) = (-x)^2 - 2(-x)$ ;  $-x > 0$   
 $\Rightarrow f(x) = x^2 + 2x$ ;  $x < 0$

$$\therefore \text{complete defn of even fn is, } f(x) = \begin{cases} x^2 - 2x; & x > 0 \\ x^2 + 2x; & x < 0 \end{cases}$$

$$(b) f(x) \text{ is odd iff } f(-x) = -f(x) \Rightarrow f(x) = -f(-x) = -x^2 - 2x, x < 0$$

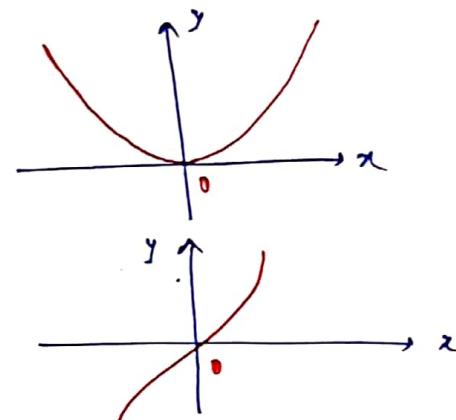
$$\therefore \text{complete defn of odd fn is, } f(x) = \begin{cases} x^2 - 2x, & x > 0 \\ -x^2 - 2x, & x < 0 \end{cases}$$



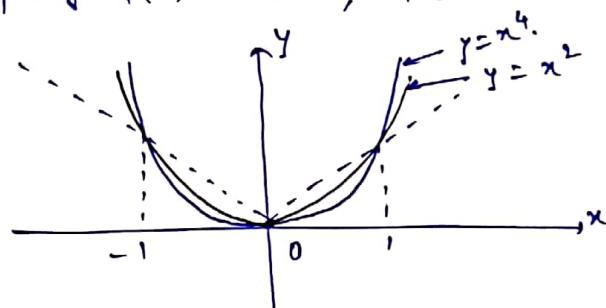
## TRANSFORMATION OF GRAPHS :-

\* Some basic graphs :-

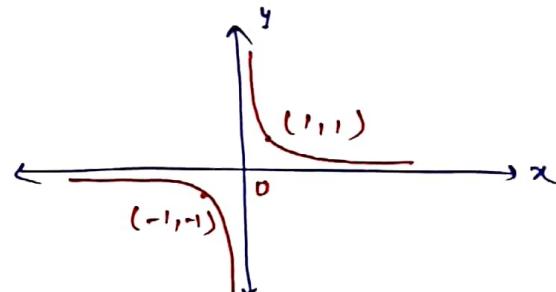
(i)  $y = x^4$



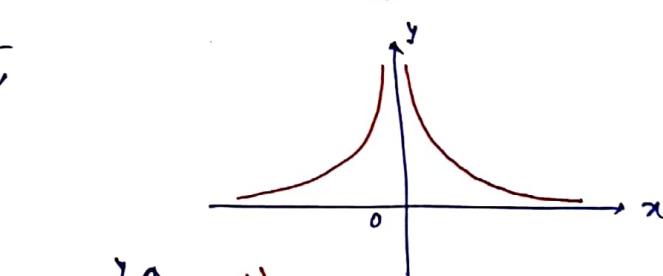
(ii)  $y = x^2$



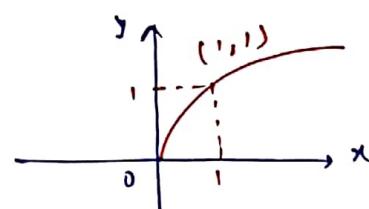
(iii)  $y = \frac{1}{x}$



Note: Graph of  $y = \frac{1}{x^2}$ ,



(iv)  $y = \sqrt{x}$



Note:  $y = \sqrt[3]{x}$ ,

