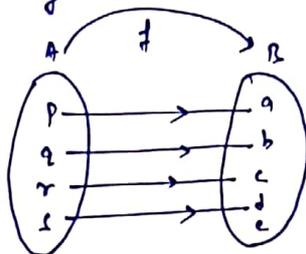


FUNCTION

Definition:-

Let A & B are two non-empty sets and 'f' be a rule, which associate each element of set A with unique element of set B ; then 'f' is called a function from A to B . Here set A is called the domain of 'f' and B is called co-domain of 'f'. The set of elements of B , which are the images of the elements of set A is called the range of 'f'.



$$f: A \rightarrow B$$

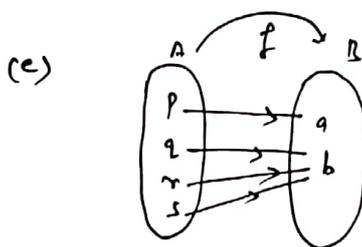
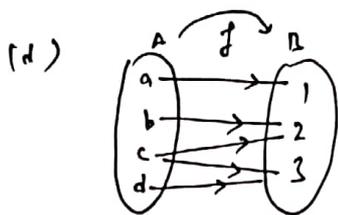
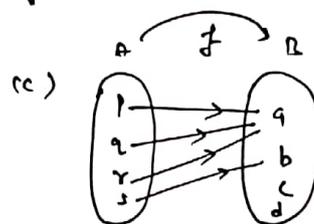
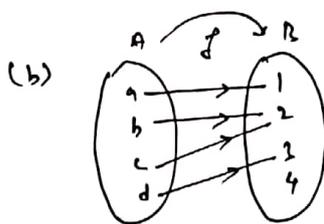
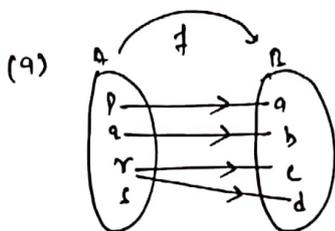
\therefore Domain of 'f': $\{p, q, r, s\}$

\therefore Co-domain of 'f': $\{a, b, c, d\}$

& Range of 'f': $\{a, b, c, d\}$

In general, Range of the funcⁿ is subset of co-domain.

1. Q. choose the functions from the following:



\therefore \therefore By defⁿ, we can say easily that (b), (c) & (e) are functions.

GRAPHICAL DEFⁿ:-

In graph of $y = f(x)$ if a st.-line parallel to y-axis is drawn, it should intersect the graph at one and only one point.

NOTE:-

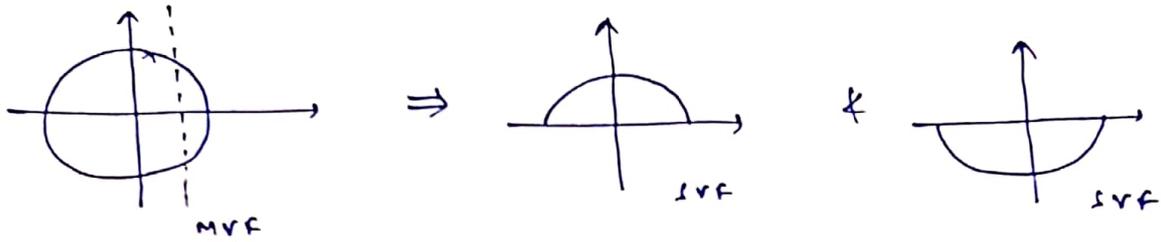
SVF :- Functions are also known as SVF (single valued f?)

MVF :- Multi-valued functions

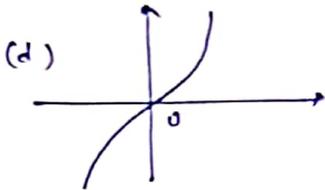
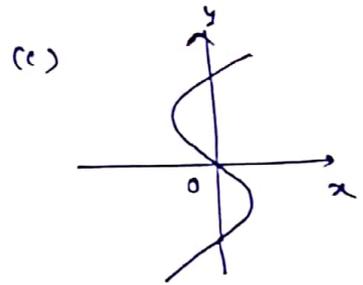
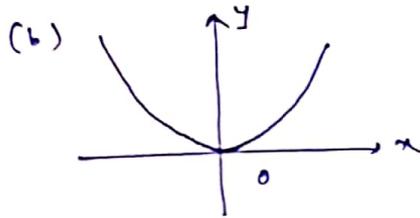
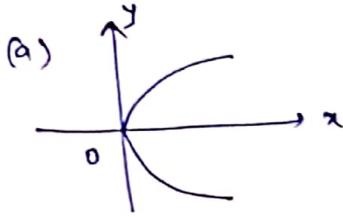
For a 2-lyte input, if there are two or more than two outputs are possible then the funcⁿ. is known as MVF.

MVF consists of atleast two SVF :-

for ex:-



2. Q. Choose funcⁿ from the following :-

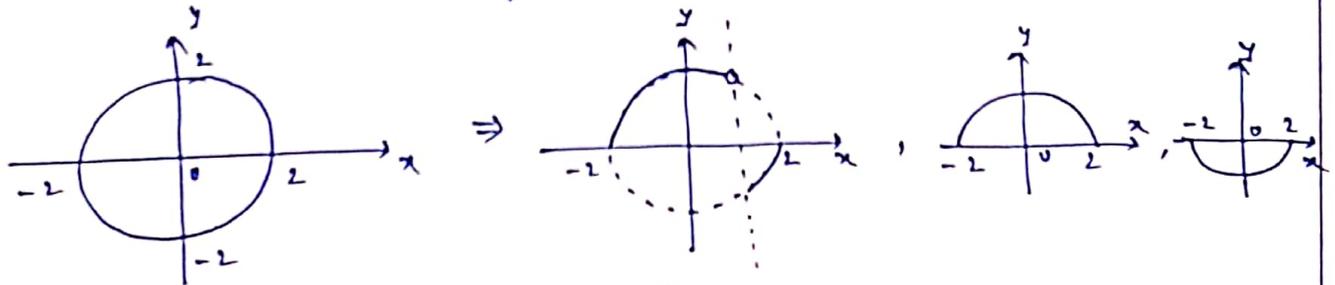


Solⁿ:- from the above graphs (b) & (d) are single valued funcⁿs. while (a) & (c) are multi-valued functions.

JEE

Q. 2. How many SVF defined in $[-2, 2]$ and satisfying $x^2 + y^2 = 4$ can be framed. How many of these are continuous?

Solⁿ:- Given eqⁿ. is eqⁿ of circle,



so, infinite funcⁿs can be framed.

But, among these functions, two are continuous. And, infinite discontⁿ funcⁿs can be framed.

4. Q. If $f(x) = \log\left(\frac{1+x}{1-x}\right)$, show that $f(x) + f(y) = f\left(\frac{x+y}{1+xy}\right)$.

→ solve it separately.

5. Q. If $f(x) = \begin{cases} 3^{x-1} & , -1 \leq x < 0 \\ |x-2| & , 0 \leq x < 1 \\ 2^{x-1} & , 1 \leq x \leq 2 \end{cases}$, find $f(2)$, $f(0)$ & $f(-1)$.

Solⁿ:-

Here, $f(2) = 2 \times 2^{-1} = 1$

$f(0) = |0-2| = 2$

$f(-1) = 3^{-2} = \frac{1}{9}$.

JEE

6. Q. If $f(x) = \cos(\ln x)$, then $f(x) \cdot f(y) - \frac{1}{2} [f(\frac{x}{y}) + f(xy)]$ has the value ---

- (a) -1 (b) $\frac{1}{2}$ (c) -2 (d) NOT.

Solⁿ:- Here, $f(\frac{x}{y}) = \cos(\ln(\frac{x}{y})) = \cos(\ln x - \ln y)$

& $f(xy) = \cos(\ln(xy)) = \cos(\ln x + \ln y)$

$\therefore f(x) \cdot f(y) - \frac{1}{2} [f(\frac{x}{y}) + f(xy)] = \cos(\ln x) \cos(\ln y) - \frac{1}{2} [\cos(\ln x - \ln y) + \cos(\ln x + \ln y)]$
 $= 0$.

So, option 'd'

JEE

7. Q. Let, $g(x)$ be a fⁿ def^d. on $[-1, 1]$. If the area of equilateral Δ with two of its vertices at $(0, 0)$ and $(x, g(x))$ is $\frac{\sqrt{3}}{4}$, then the fⁿ $g(x)$ is ---

- (a) $\pm \sqrt{1-x^2}$ (b) $\sqrt{1-x^2}$ (c) $-\sqrt{1-x^2}$ (d) $\sqrt{1+x^2}$

Solⁿ:-

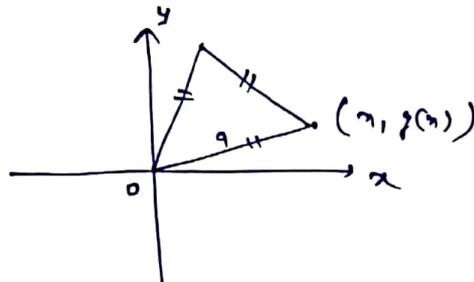
$\therefore a = \sqrt{x^2 + (g(x))^2}$

So, Area = $\frac{\sqrt{3}}{4} a^2$

= $\frac{\sqrt{3}}{4} (x^2 + (g(x))^2)$

$\therefore \frac{\sqrt{3}}{4} = \frac{\sqrt{3}}{4} (x^2 + (g(x))^2) \Rightarrow (g(x))^2 = 1-x^2 \Rightarrow g(x) = \pm \sqrt{1-x^2}$

i.e. option, (b) & (c).



JEE

8. Q. If $f: \mathbb{R} \rightarrow \mathbb{R}$ satisfies $f(x+y) = f(x) + f(y)$, for all $x, y \in \mathbb{R}$ & $f(1) = 7$ then $\sum_{r=1}^n f(r)$ is ----

- (a) $\frac{7n(n+1)}{2}$ (b) $7n/2$ (c) $\frac{7(n+1)}{2}$ (d) $7n + (n+1)$

Solⁿ:- Here, $f(x+y) = f(x) + f(y)$ & $f(1) = 7$

$$\therefore f(1+1) = 2f(1) = 2 \times 7$$

$$\therefore f(2+1) = f(2) + f(1) = 3 \times 7$$

$$f(n) = n \times 7$$

$$\text{So, } \sum_{r=1}^n f(r) = f(1) + f(2) + \dots + f(n) = 7 + 2 \times 7 + 3 \times 7 + \dots + n \times 7$$

$$= 7(1+2+3+\dots+n)$$

$$= \frac{7(n+1)n}{2}$$

option 'a'.

9. Q. If $f(x-y) = f(x)f(y) - f(a-x)f(a+y)$, 'a' is a constant, $f(0) = 1$ $f(2a-n) = \dots$

- (a) $-f(n)$ (b) $f(n)$ (c) $f(a) + f(a-x)$ (d) $f(-n)$.

Solⁿ:- putting, $x=y=0 \Rightarrow f(a) = 0$

Now, putting $x=a, y=n,$

$$f(a-n) = f(a)f(n) - f(0) \cdot f(a+n) = -f(a+n)$$

$$\text{i.e. } f(a-n) = -f(a+n)$$

putting $n = n-a$

$$\therefore f(2a-n) = -f(a+n-a) = -f(n)$$

$$\text{i.e. } f(2a-n) = -f(n)$$

So, option 'a'.

JEE

10. Q. If $f(x) = (f(x/2))^2 + (g(x/2))^2$, where $f''(x) = -f(x)$ & $g(x) = f'(x)$ and given that $f(5) = 5$, then $f(10)$ is given by ----

- (a) 5 (b) 10 (c) 0 (d) 15

solⁿ: Here, $f(x) = (f(x/2))^2 + (g(x/2))^2$

$$\begin{aligned} \therefore f'(x) &= 2f(x/2) \cdot f'(x/2) \cdot 1/2 + 2g(x/2) \cdot g'(x/2) \cdot 1/2 \\ &= f(x/2) \cdot f'(x/2) + f'(x/2) \cdot f''(x/2) \\ &= \cancel{f(x/2) \cdot f'(x/2)} - \cancel{f(x/2) \cdot f'(x/2)} \\ &= 0. \end{aligned}$$

$\therefore f(x)$ is a constant fⁿ, so, $f(10) = 5$. option a

Naming of funcⁿ:-

I. Algebraic fⁿ:-

(a) polynomial funcⁿ (rational int. fⁿ):-

funcⁿ of the form $f(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \dots + a_n$, where $a_0, a_1, a_2, \dots, a_n$ are real constants & n is non-negative integer is called a polynomial fⁿ.

ex:- $f(x) = 2x^2 + 3x + 5$, $g(x) = x + 1$ etc.

(b) rational & irrational fⁿ:-

$f(x) = P(x)/Q(x)$, $Q(x) \neq 0$ is a rational funcⁿ.

ex:- $f(x) = \frac{x+1}{x+2}$, ($x \neq -2$) ; $g(x) = \frac{x}{x^2+5}$ etc.

(c) Irrational fⁿ:-

$f(x) = \sqrt{x}$, $f(x) = \frac{x^2 + \sqrt{x}}{\sqrt{1+2x}}$ etc.

Note:- of the 3 kinds of algebraic fⁿ mentioned above don't cover all algebraic fⁿ. An algebraic fⁿ is any fⁿ $y = f(x)$ satisfying $P_0(x)y^n + P_1(x)y^{n-1} + \dots + P_n(x) = 0$, where, $P_0(x), P_1(x), P_2(x), \dots, P_n(x)$ are certain polynomials in x .

II. Transcendental funcⁿ:-

functions which are not algebraic. for ex:-

- (a) Exponential fⁿ
- (b) Logarithmic fⁿ
- (c) Trigonometric fⁿ