

Domain:-

Definition:- Domain of $f(n) = y$ is set of all real 'n' for which $f(n)$ is defined.

Rules:-

- (i) Expression under even root is always greater than or equal to '0'
- (ii) Denominator $\neq 0$
- (iii) $y = \log_a x$ is defined if $x > 0, a > 0 \& a \neq 1$.
- (iv) If domain of $f(n) + g(n)$ are D_1, D_2 resp. then the domains of $f(n) \pm g(n)$ or, $f(n) \cdot g(n)$ is $D_1 \cap D_2$.
- (v) Domain of $\frac{f(n)}{g(n)}$ is $D_1 \cap D_2 - \{g(n) = 0\}$.

(I.) Domain of Algebraic Functions:-

$$(i) f(n) = \frac{1}{\sqrt{|n| - n}}$$

say! :-
 $|n| - n > 0 \Rightarrow n \in (-\infty, 0)$

$$(ii) f(n) = \frac{1}{\sqrt{|n| - n}}$$

say! :-
 $|n| - n > 0 \Rightarrow n > n$
 $\therefore n \in \emptyset$

$$(iii) f(n) = \frac{1}{\sqrt{x - |n|}}$$

→ students must try this.

$$(v) f(n) = \sqrt{1 - \sqrt{1 - \sqrt{1 - n^2}}}$$

$$(iv) f(n) = \sqrt{\frac{1 - [n]}{4 - [n]}}$$

$$\text{say! :- } 1 - \sqrt{1 - \sqrt{1 - n^2}} > 0$$

$$\frac{1 - [n]}{4 - [n]} \geq 0$$

$$n \sqrt{1 - \sqrt{1 - n^2}} \leq 1$$

$$\therefore [n] \leq 1 \text{ or, } [n] \geq 5$$

$$n \sqrt{1 - \sqrt{1 - n^2}} \leq 1$$

$$\Rightarrow x < 2 \text{ or, } n \geq 5$$

$$n - \sqrt{1 - n^2} \leq 0$$

$$(vi) f(n) = \frac{1}{\sqrt{[n]^2 - [n] - 6}}$$

$$n \sqrt{1 - n^2} \geq 0$$

→ students must try this.

$$n - n^2 \geq 0 \Rightarrow n \in [-1, 1]$$

$$\text{Also, } 1 - n^2 \geq 0 \Rightarrow n \in [-1, 1]$$

$$\therefore n \in [-1, 1].$$

$$(vii) f(n) = \sqrt{n^2 - [n]^2}$$

$$\text{Sol}^n: n^2 - [n]^2 \geq 0$$

$$\text{or, } n^2 \geq [n]^2$$

$$\therefore n \in \mathbb{R}^+ \cup \{1\}$$

Domain Based on Logarithmic function :-

S.E.F

$$(i) y = \frac{2}{n^2 - 4} + \log_{10}(n^2 - n)$$

$$\text{Sol}^n: \text{Here, } n^2 - 4 \neq 0 \Rightarrow n \neq \pm 2 \quad \text{--- (A)} \quad \text{and } n^2 - n > 0 \Rightarrow n(n-1)(n+1) > 0$$

$$\text{so, } n \in (-1, 0) \cup (1, \infty) \quad \text{--- (B)}$$

$$\therefore A \cap B \Rightarrow n \in (-1, 0) \cup (1, 2) \cup (2, \infty)$$

$$(ii) f(n) = 4/(\log_{0.2} n)^2 + \log_{0.2} n^2 (\log_{0.2} 0.0016 n) + 26$$

$$\text{Sol}^n: \text{Let } \log_{0.2} n = t$$

$$\therefore t^2 + 2t^2 + 12t + 26 \geq 0 \Rightarrow t^2(t+2) + 12(t+2) \geq 0$$

$$\text{or, } (t^2 + 12)(t+2) \geq 0 \Rightarrow t \geq -3$$

$$\text{so, } \log_{0.2} n \geq -3 \Rightarrow n \leq (0.2)^{-3} \Rightarrow n \leq 125$$

$$\therefore n \in (-\infty, 125]$$

S.E.F

$$(iii) f(n) = \sqrt{\log_{1/2} \left(\frac{5n - n^2}{4} \right)}$$

$$\text{Sol}^n: \log_{1/2} \left(\frac{5n - n^2}{4} \right) \geq 0$$

$$\therefore 5n - n^2 > 0$$

$$\text{or, } \frac{5n - n^2}{4} \leq 1 \Rightarrow 5n - n^2 \leq 4$$

$$\text{or, } n(5-n) > 0$$

$$\text{or, } n^2 - 5n + 4 \geq 0$$

$$\text{or, } n \in (0, 1) \cup (4, \infty) \quad \text{--- (A)}$$

$$\text{or, } n^2 - 4n - n + 4 \geq 0$$

$$\text{Now, (A) } \cap \text{ (B)}$$

$$\text{or, } (n-4)(n-1) \geq 0$$

$$\therefore n \in (0, 1] \cup [4, \infty)$$

Now,

$$\frac{5n - n^2}{4} > 0$$

$$(iv) f(n) = \log_{10} (1 - \log(n^2 - 5n + 16))$$

→ Students must try this

~~SEE~~ (v) $f(n) = \frac{1}{\log_{10}(1-n)} + \sqrt{n+2}$ (Students must try this).

~~SEE~~ (vi) $2^n + 2^y = 2$

Say :- Here, $2^y = 2 - 2^n \Rightarrow y = \log_2(2 - 2^n)$

$$\therefore 2 - 2^n > 0 \Rightarrow 2^n < 2 \Rightarrow n \in (-\infty, 1)$$

Domains of Trigonometric & I.T.F functions :-

(i) $f(n) = \log x \cos x$ in $\pi_2 \leq n \leq \pi_2$

Say :- $\cos n > 0, n > 0 \& n \neq 1 \Rightarrow -\pi_2 < n < \pi_2, n > 0 \& n \neq 1$

i.e. $n \in (0, \pi_2) - \{1\}$

(ii) $\frac{1}{\sqrt{|\sin n| + \sin n}}$

Say :- $|\sin n| + \sin n > 0 \Rightarrow \sin n > 0 \Rightarrow 2n\pi < n < 2n\pi + \pi$

i.e. $n \in (2n\pi, (2n+1)\pi)$

(iii) $\sin^{-1} \{ \log_2 (\frac{1}{2}n^2) \}$

→ Students must try this.

(iv) $f(n) = \sin^{-1} \left(\frac{1+n^2}{2x^{3/2}} \right) + \sqrt{\sin(\sin n)} + \log_{(3f(n)+1)}(n^2+1)$

Say :- for, $\log_{(3f(n)+1)}(n^2+1) \rightarrow f(n) \neq 0 \Rightarrow x \neq 1 \dots (A)$

& $\sin(\sin n) \geq 0 \Rightarrow 0 \leq \sin n \leq 1$

& i.e. $n \in [2n\pi, (2n+1)\pi], n \in \mathbb{Z} \dots (B)$

$$-1 \leq \frac{1+n^2}{2x^{3/2}} \leq 1$$

$$\Rightarrow -1 \leq \frac{x^{3/2} + n^{3/2}}{2} \leq 1, \text{ But } \frac{n^{-3/2} + n^{3/2}}{2} \geq 1 \quad (\text{As } n > 0)$$

$$\therefore \frac{n^{-3/2} + n^{3/2}}{2} = 1 \Rightarrow n^{-3/2} = x^{3/2} = 1 \Rightarrow n^3 = 1 \dots (C)$$

Hence, (A) \cap (B) \cap (C)

i.e. $n \in \emptyset$

$$(v) f(n) = \cos^{-1} \left(\frac{2-n}{4} \right) + [\log(2-n)]^{-1}$$

Soln:- $\frac{1}{\log(2-n)}$ is defined if $\log(2-n) \neq 0$ & $2-n > 0$

$$\Rightarrow 2-n \neq 1 \text{ & } n < 2$$

$$\Rightarrow n \neq 1 \text{ & } n < 2 \quad \dots \dots \textcircled{A}$$

$$\text{Also, } -1 \leq \frac{2-n}{4} \leq 1 \Rightarrow -4 \leq 2-n \leq 4 \Rightarrow -6 \leq -n \leq 2$$

$$\text{i.e. } -2 \leq n \leq 6 \Rightarrow n \in [-6, 6] \quad \dots \dots \textcircled{B}$$

$$\therefore \textcircled{A} \wedge \textcircled{B}$$

$$\text{i.e. } n \in [-6, 6] - \{2\}$$

$$(vi) f(n) = \cos^{-1} \left(\frac{3}{4+2\sin n} \right)$$

$$\text{Soln:- } -1 \leq \frac{3}{4+2\sin n} \leq 1 \Rightarrow 4+2\sin n > 0 \quad \forall n \in \mathbb{R}$$

$$\therefore \frac{3}{4+2\sin n} \leq 1$$

$$\Rightarrow 4+2\sin n \geq 3 \Rightarrow \sin n \geq -\frac{1}{2}$$

$$\text{i.e. } n \in \left[-\pi_6 + 2n\pi, \frac{7\pi}{6} + 2n\pi \right], n \in \mathbb{Z}$$

$$(vii) f(n) = \tan^{-1} \sqrt{n(n+1)} + \sin^{-1} (\sqrt{n^2+n+1}).$$

$$\text{Soln:- } n(n+1) \geq 0 \Rightarrow n \leq -1 \text{ or } n \geq 0 \quad \dots \dots \textcircled{A}$$

$$0 \leq n^2+n+1 \leq 1 \Rightarrow n^2+n+1 \leq 1 \quad (n^2+n+1 > 0 \quad \forall n \in \mathbb{R})$$

$$\text{So, } -1 \leq n \leq 0 \quad \dots \dots \textcircled{B}$$

$$\text{So, } \textcircled{A} \wedge \textcircled{B}$$

$$\text{i.e. } n \in \{-1, 0\}$$

$$(viii) f(n) = \sin^{-1} \left(\frac{\sin n}{n} \right)$$

$$(ix) f(n) = \sqrt{n^2+n-1}$$

$$(ix) f(n) = \sqrt{\cos(\sin n)} + \sqrt{\log_n f(n)}$$

$$(x) f(n) = \sqrt{\sin^{-1} 2n + \pi/6}$$

→ students must try above four questions also.

Some important questions on Domain :-

(1.) If $f(\sin x) = \frac{(2 \tan x + \sec^2 x)(1 + \cos 2x)}{2}$ then determine the domain & range of $f(t)$.

Soln:- $\therefore f(\sin x) = 1 + \sin 2x$

$$\therefore f(t) = 1 + t, -1 \leq t \leq 1$$

i.e. $t \in [-1, 1]$

(2.) Domain of $f(n) = \sqrt{n^2 + 4n + 2}$

$$\text{Soln:- } n(n+4) \geq 0 \Rightarrow n \leq -4, n \geq 0$$

$$n^2 + 4n \geq 2n^2 + 2 \Rightarrow n \in [1, 2]$$

$$\therefore n = \{1, 2, 3\}$$

(3.) If domain of $y = f(n)$ is $-3 \leq n \leq 2$, what is the domain of $g(n) = f\{1[n]\}$.

Soln:- $f\{1[n]\}$ has domain given by $-3 \leq 1[n] \leq 2$

$$\Rightarrow \begin{cases} -3 \leq 1[n] \\ 1[n] \leq 2 \end{cases} \Rightarrow \begin{cases} n \in \mathbb{R} \\ -2 \leq n \leq 2 \end{cases} \Rightarrow \begin{cases} n \in \mathbb{R} \\ -2 \leq n < 3 \end{cases}$$

$$\therefore n \in [-2, 3)$$

(4.) $f: [-4, 4] \rightarrow \{-\pi, 0, \pi\} \rightarrow \mathbb{R}$, where $f(x) = \cot(\sin x) + \left[\frac{n^2}{|a|}\right]$

is ~~not~~ an odd f ? complete set of values of 'a' is ...

(a) $(-16, 16) - \{0\}$ (b) $(-\infty, -16) \cup (16, \infty)$ (c) $[-16, 16] - \{0\}$

(d) $(-\infty, -16] \cup [16, \infty)$

Soln:- for $f(n)$ to be odd, $\left[\frac{n^2}{|a|}\right]$ shouldn't depend upon the value of x

$$\therefore n \in [-4, 4] \Rightarrow 0 \leq n^2 \leq 16$$

$$\Rightarrow \left[\frac{n^2}{|a|}\right] = 0, \text{ if } |a| > 16$$

$$\text{i.e. } a \in (-\infty, -16) \cup (16, \infty)$$

$$\therefore \boxed{\text{option 'b'}}$$