

PT-2/HALF YEARLY EXAMINATION, 2022-23
APPLIED MATHEMATICS

Time – 3 hrs.

Class – XII

M.M. : 80

Date – 12.09.2022 (Monday)

Name of the student _____ Section _____

General Instructions:

- This question paper contains two **parts A and B**.
- **Part – A:**
- It consists of three sections- **I, II and III**.
- Section **I** comprises of 7 MCQs type questions of **1 Mark** each.
- Section **II** comprises of 6 MCQs type questions of **2 Marks** each.
- Section **III** case study MCQs of **1 Mark each**.
- **Part – B:**
- It consists of three sections- **IV, V and VI**.
- Section **IV** comprises of 8 questions of **2 marks** each.
- Section **V** comprises of 4 questions of **4 marks** each.
- Section **VI** comprises of 4 questions of **6 marks** each.

Part-A

Section I : Comprises of 7 MCQs type questions of 1 Mark each

- Q1 $(18x + 10) \pmod{7}$ is 1
(a) 5 (b) 4 (c) 3 (d) 2
- Q2 A boat running downstream covers a distance of 16 km in 2 hours while for covering the same distance upstream it takes 4 hours. What is the speed of the boat in still water? 1
(a) 4 km/h (b) 6 km/h (c) 8 km/h (d) 10 km/h
- Q3 The solution set of $6 \leq -3(2x-4) < 12, x \in \mathbb{R}$ is 1
(a) $[0, 1]$ (b) $[1, 0)$ (c) $(0, 1)$ (d) $[0, 1]$
- Q4 If $y = Ae^{5x} + Be^{-5x}$, then d^2y/dx^2 is equal to 1
(a) 25y (b) 5y (c) -25y (d) 15y
- Q5 If $y = x(x-3)^2$ strictly decreases for the values of x given by 1
(a) $1 < x < 3$ (b) $x < 0$ (c) $x > 0$ (d) $0 < x < 3/2$
- Q6 The slope of the normal to the curve $x = t^2 + 3t - 8$; $y = 2t^2 - 2t - 5$, at $t = 2$ is 1
(a) -7/6 (b) 6/7 (c) 1 (d) 5/6

- Q7 A die is rolled thrice. If the event of getting an even number is a success, then the probability of getting at least two successes is 1
 (a) $7/8$ (b) $3/8$ (c) $2/3$ (d) $1/2$

Section II : Comprises of 6 MCQs type questions of 2 Marks each.

- Q8 Three types of wheat costing Rs. 18 per kg, Rs. 20 per kg and Rs. 25 per kg are mixed together. If the mixed variety is sold at Rs. 22 per kg, then the ratio in which these types of wheat should be mixed respectively is 2
 (a) 1:2:3 (b) 2:2:3 (c) 2:3:1 (d) 1:1:2
- Q9 Two pipes A and B together can fill a tank in 4 hours. Pipe B take 6 hours more than A to fill the tank, if they opened separately. The time taken by A to fill the tank alone is 2
 (a) 2 hours (b) 4 hours (c) 6 hours (d) 8 hours.
- Q10 A box contains 100 bulbs of which 10 are defective. The probability that out of a sample of 5 bulbs drawn one by one with replacement none is defective is 2
 (a) $\left(\frac{1}{2}\right)^5$ (b) $\frac{9}{10}$ (c) $\left(\frac{9}{10}\right)^5$ (d) $\left(\frac{1}{10}\right)^5$
- Q11 The total revenue received from the sale of x units of a product is given by $R(x) = 3x^2 + 6x + 5$. The marginal revenue, when $x = 5$ is 2
 (a) 36 (b) 35 (c) 34 (d) 33
- Q12 Evaluate the integral $\int_1^e \frac{dx}{x(1 + \log x)^2}$ 2
 (a) 0 (b) $1/2$ (c) $\sqrt{2}$ (d) $\sqrt{2} - 1$
- Q13 If A is a square matrix such that $A^2 = A$, then $(I - A)^3 + A$ is equal to 2
 (a) 0 (b) A (c) $I - A$ (d) I

Section III: Case study MCQs of 1 Mark each

- Q14 A farmer has a piece of land. He observed that he got 600 units of fruits per tree by planting up to 25 trees and when 26 trees were grown, he received 15210 units of fruits, for 27 trees he ended up with 15390 fruits, for 28 trees he got 15540 fruits and this sequence of production of fruits continues in the same pattern as more trees, in excess of 25, were grown. 5

Based on the above information answer the following questions:

- i) If 'x' more trees, in excess of 25 are grown, then the number of fruits produced per tree is
 a) $600 - 15x$ b) $600 + 15x$ c) $600x - 15$ d) $600x + 15$
- ii) The production of entire garden if 'x' more trees, in excess of 25, are planted
 a) $(25 + x)(600 + 15x)$ b) $(25 - x)(600 - 15x)$
 c) $(25 + x)(600 - 15x)$ d) $(25 + x)(15x - 600)$

- iii) The marginal production of the garden when 'x' more trees, in excess of 25, are planted
- a) $225 + 30x$ b) $225 - 30x$ c) $225x + 30$ d) $225x - 30$
- iv) The critical point of producing 'x' more units of trees is
- a) 7 b) 8 c) 7.5 d) 8.5
- v) The number of trees to be grown to get maximum production is
- a) 30 or 31 trees b) 32 or 33 trees
c) 33 or 34 trees d) 34 or 35 trees

Part-B

Section IV: Comprises of 8 questions of 2 marks each.

- Q15 Express the following matrix as the sum of a symmetric matrix and a skew symmetric matrix. 2

$$A = \begin{bmatrix} 1 & 3 & 5 \\ -6 & 8 & 3 \\ 4 & 6 & 5 \end{bmatrix}$$

OR

Find the value of x from the following matrix equation:

$$\begin{bmatrix} 1 & x & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 1 \\ 15 & 3 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix} = 0.$$

- Q16 Find the value of x and y from the following matrix equation: 2

$$\begin{bmatrix} x-y & z \\ 2x-y & w \end{bmatrix} = \begin{bmatrix} -1 & 4 \\ 0 & 5 \end{bmatrix}$$

- Q17 What positive values of x makes the following pair of determinants are equal 2

$$\begin{vmatrix} 2x & 3 \\ 5 & x \end{vmatrix} = \begin{vmatrix} 16 & 3 \\ 5 & 2 \end{vmatrix}$$

- Q18 Solve the differential equation $xy \frac{dy}{dx} = (x+2)(y+2)$; $y = -1$ when $x = 1$. 2

- Q19 Evaluate the integral $\int e^x \left(\frac{1}{x^2} - \frac{2}{x^3} \right) dx$. **OR** $\int \log x dx$. 2

- Q20 Evaluate the integral $\int_2^3 \frac{1}{(x+1)(x+2)} dx$ 2

- Q21 Find all the positive integers less than 30 forming the equivalence class of 5 for modulo 7. 2

OR

Find the remainder when $862 \times 783 \times 671 \times 549$ is divided by 8.

- Q22 Priya can run 1 km in 3 min 10 sec and Preeti can cover the same distance in 3 min 20 sec. By what distance can Priya beat Preeti? 2

Section V: Comprises of 4 questions of 4 marks each.

- Q23 In a 10 km race, A, B and C each running at uniform speed get the gold, silver and bronze medals respectively. If A beats B by 1 km and B beats C by 1 km, then find by how many metres does A beat C? 4

OR

A boat covers 32 km upstream and 36 km downstream in 7 hours. Also, it covers 40 km upstream and 48 km downstream in 9 hours. Find the speed of the boat in still water and that of the stream.

- Q24 In 3 trials of a binomial distribution, the probability of 2 successes is 9 times the probability of 3 successes. Find the probability of success in each trial. 4

OR

A class XII has 20 students whose marks (out of 30) are 14, 17, 25, 14, 21, 17, 17, 19, 18, 26, 18, 17, 17, 26, 19, 21, 21, 25, 14 and 19. If random variable X denotes the marks of a selected student given that the probability of each student to be selected is equally likely. Find mean, and variance of the probability distribution.

- Q25 The demand and supply functions are $P_d = 25 - x^2$ and $P_s = 2x + 1$ respectively. Find the consumer's surplus and producer's surplus at equilibrium price. 4

- Q26 If $x\sqrt{1+y} + y\sqrt{1+x} = 0$, then find $\frac{dy}{dx}$. 4

Section VI: Comprises of 4 questions of 6 marks each

- Q27 Using cramer's rule, solve the following equations: 6
 $x - 2y + z = 3; \quad 2x + y - z = 5; \quad 3x - y + 2z = 12$

- Q28 If matrix $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, then Prove that $A^2 - 4A - 5I = 0$, using it find A^{-1} . 6

OR

Solve the following system of equation using matrix method

$$x + y + z = 10, \quad 2x + y = 13, \quad x + y - 4z = 0$$

- Q29 In a bank, principal increases continuously at the rate of r% per year. Find the value of r if Rs100 double itself in 10 years, using differential equation. ($\log_e 2 = 0.6931$) 6

- Q30 Find the mean and variance of probability distribution of number of heads in three tosses of a fair coin. 6

OR

Four bad eggs are mixed with 10 good ones. If three eggs are drawn one by one with replacement, then find the probability distribution of the number of bad eggs drawn.



PT-2/HALF YEARLY EXAMINATION, 2022-23
MATHEMATICS STANDARD

Time – 3 hrs.

Class – XII

M.M. : 80

Date – 12.09.2022 (Monday)

Name of the student _____ Section _____

General Instructions:

- This question paper contains **six sections**. Each section is compulsory.
- **Section A**-7 Multiple Choice Questions (each of 1 mark)
- **Section B**-6 Multiple Choice Questions (each of 2 marks)
- **Section C**- 8 Subjective Type Questions (each of 2 marks)
- **Section D**- 4 Subjective Type Questions (each of 4 marks)
- **Section E**- 4 Subjective Type Questions (each of 6 marks)
- **Section F**- 1 Case Study based Question (5 marks)

Section-A

Multiple Choice Questions (1 mark each)

- Q1. The principal value of $\tan^{-1}(\tan 3\pi/5)$ is
(a) $2\pi/5$ (b) $-2\pi/5$ (c) $3\pi/5$ (d) $-3\pi/5$
- Q2. $\sin[\pi/3 - \sin^{-1}(-1/2)]$ is equal to:
(a) $1/2$ (b) $1/3$ (c) -1 (d) 1
- Q3. Value of k, for which $A = \begin{bmatrix} k & 8 \\ 4 & 2k \end{bmatrix}$ is a singular matrix is -
(a) 4 (b) -4 (c) ± 4 (d) 0
- Q4. Value of x if $\begin{bmatrix} 2 & 2x+3 \\ 3x-2 & 2 \end{bmatrix}$ is a symmetric matrix.
a) 3 b) 4 c) 5 d) 6
- Q5. The value of $\begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix}$ will be equal to
A) $a^3+b^3+c^3$ B) $3abc$ C) $a^3+b^3+c^3 -3abc$ D) 0
- Q6. If $\begin{vmatrix} 2x & -1 \\ 4 & 2 \end{vmatrix} = \begin{vmatrix} 3 & 0 \\ 2 & 1 \end{vmatrix}$ then x is
A) 3 B) $2/3$ C) $3/2$ D) $-1/4$

- Q7. If $\int \sec^2 (7 - 4x)dx = a \tan (7 - 4x) + C$, then value of a is
 a) -4 b) $-\frac{1}{4}$ c) 3 d) 7

Section-B

Multiple Choice Questions (2 marks each)

- Q8. If $\sin^{-1}x + \sin^{-1}y = \pi/2$, then value of $\cos^{-1}x + \cos^{-1}y$ is
 (a) $\pi/2$ (b) π (c) 0 (d) $2\pi/3$
- Q9. The value of the expression $\sin [\cot^{-1} (\cos (\tan^{-1} 1))]$ is
 (a) 0 (b) 1 (c) $1/\sqrt{3}$ (d) $\sqrt{(2/3)}$
- Q10. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = x^2 + 1$. Then, pre-images of 17 and -3 , respectively, are
 (a) $\varphi, \{4, -4\}$ (b) $\{3, -3\}, \varphi$ (c) $\{4, -4\}, \varphi$ (d) $\{4, -4\}, \{2, -2\}$
- Q11. If A(3,4) B(-7, 2) and C(x, y) are collinear then
 A) $x+5y+17 = 0$ B) $x+5y+13 = 0$ C) $x-5y+17 = 0$ D) None of these
- Q12. Given that $x = at^2$ and $y = 2at$, then value of $\frac{d^2y}{dx^2}$ is
 a) $-\frac{1}{2at^3}$ b) $-\frac{1}{2at^2}$ c) $\frac{1}{t^2}$ d) $\frac{-2a}{t}$
- Q13. $\int (\sin^2x - \cos^2x) / \sin^2x \cos^2x dx$ is equal to
 (a) $\tan x + \cos x + c$ (b) $\tan x + \operatorname{cosec} x + c$ (c) $\tan x + \cot x + c$ (d) $\tan x + \sec x + c$

SECTION - C

Subjective Type Questions (each of 2 marks)

- Q14. Show that function $f: \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = 2x+3$ is one-one onto.
- Q15. Differentiate x^x w.r.t x .

OR

Find dy/dx if $y + \sin y = \cos x$

- Q16. If $y = 3e^{2x} + 2e^{3x}$, prove that $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$

OR

If $y = A \sin x + B \cos x$, then prove that $d^2y/dx^2 + y = 0$

- Q17. The volume of a cube is increasing at the rate of $8 \text{ cm}^3/\text{s}$. How fast is the surface area increasing when the length of an edge is 12 cm?
- Q18. A ladder 5 m long is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall, at the rate of 2 m/s. How fast is its height on the wall decreasing when the foot of the ladder is 4 m away from the wall ?

OR

A man of height 2 metres walks at a uniform speed of 5 km/h away from a lamp post which is 6 metres high. Find the rate at which the length of his shadow increases.

Q19. Solve :

$$\int \frac{x^3 + 3x + 4}{\sqrt{x}} dx$$

Q20. Solve :

$$\int \frac{1}{1 + \tan x} dx$$

Q21. Solve :

$$\int \sin^3 x \cos^2 x dx$$

OR

Integrate w.r.t x:

$$\frac{\tan^4 \sqrt{x} \sec^2 \sqrt{x}}{\sqrt{x}}$$

SECTION - D

Subjective Type Questions (4 marks each)

Q22. Express $\begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix}$ as a sum of a symmetric and a skew symmetric matrix.

Q23. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$ then prove that $(AB)^T = B^T A^T$.

Q24. Find k if f(x) is continuous

$$f(x) = \begin{cases} \frac{k \cos x}{\pi - 2x}, & \text{if } x \neq \frac{\pi}{2} \\ 3, & \text{if } x = \frac{\pi}{2} \end{cases} \quad \text{at } x = \frac{\pi}{2}$$

OR

Find values of a & b if f(x) is continuous at x= 2 and x=10

$$f(x) = \begin{cases} 5, & \text{if } x \leq 2 \\ ax + b, & \text{if } 2 < x < 10 \\ 21, & \text{if } x \geq 10 \end{cases}$$

Q25. Find the intervals in which the function f given by $f(x) = 4x^3 - 6x^2 - 72x + 30$ is
(a) increasing (b) decreasing.

OR

Find local maximum and local minimum values of the function f given by

$$f(x) = 3x^4 + 4x^3 - 12x^2 + 12$$

SECTION - E

Subjective Type Questions (6 marks each)

- Q26. Show that $R = \{ (a,b) : |a-b| \text{ is divisible by } 2 \text{ where } a, b \text{ are integers} \}$ is an equivalence relation.
- Q27. Solve using inverse of matrices –
 $5x+3y+z= 16, 2x+y+3z= 19, x+2y+4z= 25$
- Q28. If $\cos y = x \cos (a+y)$, with $\cos a \neq \pm 1$ then prove that $\frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$

OR

Differentiate w.r.t x : $\cos x \cdot \cos 2x \cdot \cos 3x$

- Q29. Show that the maximum volume of a cone inscribed in a sphere of radius R cm is $\frac{8}{27}$ times of the volume of the sphere.

OR

Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$. Also find the maximum volume of the cylinder.

SECTION - F

Case Study based Question (5 marks)

- Q.30 Megha wants to prepare a handmade gift box for her friend's birthday at home. For making lower part of the box, she takes a square piece of cardboard of side 20 cm.



Based on the above information, answer the following questions.

- (i) If x cm be the length of each side of the square cardboard which is to be cut off from corners of the square piece of side 20cm, then possible value of x will be given by the interval :
- (a) $[0, 20]$ (b) $(0, 10)$ (c) $(0, 3)$ (d) none of these
- (ii) Volume of the open box formed by folding up the cutting corners can be expressed as
- (a) $V = x(20 - 2x)(20 - 2x)$ (b) $V = x/2(20 + x)(20 - x)$
(c) $V = x/3(20 - 2x)(20 + 2x)$ (d) $V = x(20 - 2x)(20 - x)$
- (iii) The values of x for which $\frac{dV}{dx} = 0$, are
- (a) 3, 4 (b) 0, 10/3 (c) 0, 10 (d) 10, 10/3
- (iv) Megha is interested in maximising the volume of the box, so, what should be the side of the square to be cut off so that the volume of the box is maximum ?
- (a) 12 cm (b) 8 cm (c) 10/3 cm (d) 2 cm
- (v) The maximum value of the volume of the box is
- (a) $\frac{17000}{27} \text{ cm}^3$ (b) $\frac{11000}{27} \text{ cm}^3$ (c) $\frac{8000}{27} \text{ cm}^3$ (d) $\frac{16000}{27} \text{ cm}^3$

